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THE EQUATIONS OF A MAGNETOHYDRODYNAMIC ALTERNATING-CURRENT GENERATOR HAVING A ROTATING MAGNETIC FIELD

Ye. I. Yantovskiy

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In a number of articles [1 through 4] on the working of magnetohydrodynamic (MHD) generators, the lay out of a high-power (on the order of hundreds of megawatts) conduction [5] a. c. generator is presented, and the results of experiments on models with power up to 205 kw are given [2].

There is also a suggestion on the use of a conduction generator with a power of 10 million kw in combination with a gaseous nuclear reactor [6].

In these generators it is difficult to ensure sufficient electrode life (especially in an oxidizing atmosphere) and it is impossible to obtain voltages in excess of several thousand volts.

The conversion of direct current into alternating in order to increase the voltage to hundreds of thousands of volts considerably complicates the entire system and lowers the advantages obtainable by direct conversion.

Therefore the main direction of the development of power-station engineering is to us the direct conversion of energy from gas flow

into electrical energy in the form of alternating current in an electrodeless MHD generator with a rotating magnetic field. Alternating current from this generator can have a very high voltage, which makes it possible to dispense with step-up transformers. The power of the generator is not limited by structure, is determined only by the delivery of the working body, and can be tens of millions of kilowatts per unit.

The schematics of generators having a recitilinearly traveling field and a radially traveling field in accordance with the direction of the gas flow have also been examined [7 and 8], however, of most interest are layouts with a rotating magnetic field and a channel with a circular cross section.

L. A. Simonov turned our attention to the possibility of creating vortex gas flows at supersonic speeds in blade-guide apparatus, with the aim of using them in generators with a rotating field.

Aside from asynchronous generators, which contain no rotating parts, generators with a rotating rotor or magnetic turbines are also of technical interest. Their theory, concerning gas flow, is in no way different from the theory of asynchronous generators.

The following symbols are used in the article:

A--mechanical equivalent of thermal energy,

b--length of the flow region before the magnetic circuit,

c--length of the flow region after the magnetic circuit,

G_p--thermal capacity at constant pressure,

D--fluid diffusion of energy,

E--electrical field,

e--charge of an electron,

F--magnetizing force forming the external magnetic field,

f_B--force of viscosity applied to a unit volume.

G--gas (liquid) flow rate per unit of width of the channel (along circumference),

H--strength of external magnetic field,

H--effective magnetic field (total),

h--strength of natural magnetic field,

i--enthalpy,

I--current internal electric circuit,

J--mechanical equivalent of electrical energy.

j--current density in the gas (liquid),

$k = C_p/C_v$--index of adiabatic line,

$\underline{l}$--length of magnetic circuit forming the walls of channel,

n_e--electron density,

P--power,

p--pressure,

Q--heat liberation per unit volume per unit time,

q--specific heat flow to the wall,

r--radius of channel in transverse cross section,

s--slippage of conducting medium relative to the external magnetic field,

T--true temperature of conducting medium,

t--time,

U--phase voltage of external circuit,

V--rate of motion of conducting medium,

$V_s = \omega/\alpha$--turning rate of external magnetic field,

w--number of turns,

x, y, z, r, ϑ --space coordinates,

$\alpha = \pi/\tau$ --wave number of the external magnetic field,

δ--the distance between the walls of the channel (magnitude of the gap)--dimension of flow in direction,

ζ--coefficient of surface friction,

η--efficiency,

λ--thermal conductivity of conducting medium

μ--permeability,

ρ--density of conducting medium,

σ--specific electrical conductivity of the conducting medium,

σ_0--electrical conductivity in the absence of a magnetic field,

τ--pole separation, one-half of the space period of the magnetic field,

τ_W--stress of surface friction,

φ--phase shift angle between voltage and current in external circuit,

ψ--phase shift angle between natural and external magnetic fields

ω--angular frequency of the external field.

Subscripts: 1--initial value; 2--final value; 3--active component; r--reactive component; m--maximum (amplitudinal) value; x, y, z, r, ϑ--components along the coordinate axes; 0--decelerated parameters.

Setting up the problem. The flow scheme is shown in Fig. 1 An electrically conductive medium (gas) flows into a channel with a circular cross section whose walls have infinite permeability. In the electrically non-conductive walls of the channel are mounted leads with three-phase alternating current.

The rotating magnetic field of this current has a turning rate which differs from the initial velocity of gas. Depending upon the

sign of the difference between them, either deceleration or acceleration of the gas flow takes place owing to the interaction of the currents in the gas with the magnetic field, that is, the separation of energy from the gas flow or connection of the energy to it (asynchronous MHD generator or motor).

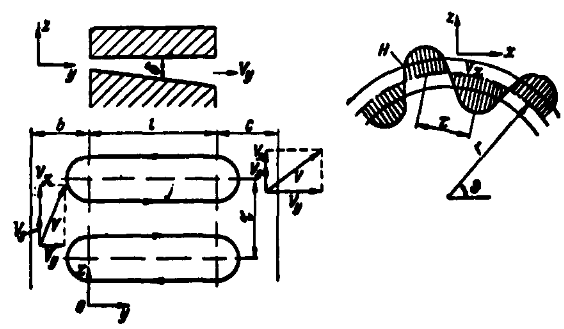

Fig. 1.

The rotating magnetic field may be created also by a rotating magnetizing rotor with direct-current electromagnets or permanent magnets. In both cases of the creation of an external rotating field, it distribution remains the same and the stationary flow of the gas is not a function of the methods of creating the external field. With a rotating rotor the energy of the gas flow is transformed into electrical energy if the magnetic circuit--stator--has leads connected to a circuit (synchronous MHD generator) or into mechanical energy of the turning of a rotor (magnetic turbine).

These considerations pertain to an asynchronous MHD generator, but may easily be applied to rotating machines.

The effects shown in Fig. 1 in cross section s, z correspond to an ordinary alternating-current electric machine. These same effects, shown in Fig. 1 in the cross section y, z, may be treated as the work-

ing process of an active turbine.

From the point of view of the theory of the usual electric apparatus, a MHD generator with a rotating field may be treated as the limit of an infinitely large number of infinitely short ordinary generators distributed on a single axis, each of which has its rotor velocity, momentum and slippage at an over-all magnetizing current for each and the same velocity of the rotating field.

Similarly, in the usual asynchronous machine the rotor speed is determined by dynamics equations, and in the individual case of steady-state motion it is determined by the equality of the momentum and moment of drag. The stationary operating conditions of magnetohydrodynamic apparatus are determined by hydrodynamic equations.

Although the effects occurring in each transverse cross section of MHD apparatus do not differ from those in the usual electrical apparatus, during transition from one cross section to another the ability of the "rotor" to change its shape and volume manifests itself, and this uncovers possibilities of uniting gas and electrical machine into a single unit.

The flow scheme in Fig. 1 may be examined also from the point of view of ordinary turbines as the streamlining of blading made up of "magnetic blades." At its input and output it is possible to set up the usual triangles of velocities for turbines (Fig. 1, cross section x, y).

A decrease in the peripheral velocity of the flow while passing through the region of the rotating field corresponds to the usual active turbine.

In the first, very rough approximation a representation of the magnetic flux lines has been put forth by Ustimenko and myself earlier [9].

In the present setting up of the problem, let us assume that clearance is considerably less than the length of the magnetic circuit and the pole separation, and the dispersion of the magnetic field can be ignored.

Initial equations. Let us use the general equations of an electromagnetic field and hydrohynamics in the MKSA system.

Maxwell's equations

$$\mathbf{curl}\ \mathrm{H} = \mathrm{j} \tag{1}$$

$$\mathbf{curl}\ \mathrm{j} = \mu\sigma\left(\mathbf{curl}\, \mathrm{V} \times \mathrm{H} - \frac{\partial \mathrm{H}}{\partial t}\right) \tag{2}$$

$$\mathrm{div}\ \mathrm{H} = 0 \tag{3}$$

In Equation (2) Ohm's law was used for the isotropic conducting medium

$$\mathrm{j} = \sigma(\mathrm{E} + \mathrm{V} \times \mu\mathrm{H}) \tag{4}$$

The assumption about the isotropy of the medium is made for simplicity, but in reality the tensor nature of σ may fully manifest itself. The characteristic parameter determining this effect is the product of the Larmor electron frequency and the average time between the collision of an electron with neutrals and ions

$$\omega_e \tau_e = \frac{\mu H}{n_e e} \sigma_0 \tag{5}$$

is not always small in comparison with unity, therefore, conductivity across the field

$$\sigma_\perp = \frac{\sigma_0}{1 + (\omega_e \tau_e)^2} \tag{6}$$

may be lower than that indicated earlier in the graphs of σ_0. In a mixture of 1% potash with air at a pressure of 1 atm (tech.), T = = 3000°K, μ H = 10000 gs.

$$\sigma_0 = \frac{1}{\text{Ом см}}, \qquad \omega_e \tau_e = 0.8$$

The continuity equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{V}) = 0 \tag{7}$$

The momentum equation

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho (\mathbf{V}\nabla)\,\mathbf{V} = -\nabla p + J\mathbf{j} \times \mu \mathbf{H} + \mathbf{f}_{\mathrm{B}} \tag{8}$$

Here $J_j \times \mu H$ is the electrical space force (ESF).

The energy equation [10]

$$\rho \frac{di_0}{dt} = \frac{\partial p}{\partial t} + D + AQ + J\frac{j^2}{\sigma} + J\mathbf{V}\cdot(\mathbf{j}\times\mu\mathbf{H}) + A\lambda\nabla^2 T \tag{9}$$

Here j^2/σ is the Joule effect per unit volume, and $V \cdot (j \times \mu H)$ is the power of the ESF.

The equation of the state of an ideal gas, correct at a low degree of dissociation and ionization, is

$$p = \rho RT \tag{10}$$

The essential factor affecting the nature of the flow is the function σ (p, T).

Heated gases with admixtures of alkali metals are used as the working bodies in MHD machines. The calculated dependences $\sigma_0(p, T)$ are shown in Fig. 2 for several mixtures.

The degree of ionization of the admixtures was determined by the Saha formula, and the electrical conductivity σ_0 as the average of values obtained by formulas for weakly and strongly ionized gases (i.e., for weakly ionized gases in a negligible cross section of collision with ions and for strongly ionized gases in a negligible cross section with neutrals [11]).

Because of the insufficiently known collision cross sections of low-energy electrons ($\sim$ 0.2 electron volts) with neutrals and a cer-

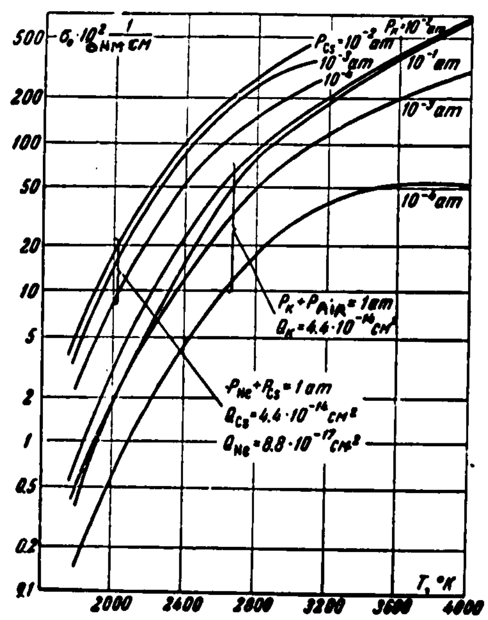

Fig. 2.

(am = atm (tech))

tain principal indeterminancy of the theory, σ cannot be calculated with high accuracy.

The existing meager experimenal data pertaining to σ_0 in general supports that shown in the graphs.

Regarding the complexity of the formula $\sigma(p, T)$, solutions of equations taking this dependence into account are advantageously carried out numerically on computers, taking the additional terms in (2) into account.

In analytic investigations it is assumed that σ = const, which is fully permissible in the temperature range where total ionization of the admixtures is attained, but is only a rough approximation at lower temperatures.

The basis system of equations of a MHD machine with a rotating field. Let us transform the system of initial equations.

Inasmuch as $\delta \ll \tau, \delta \ll l, \delta \ll r$, it may be assumed that the magnetic field in the channel has only a z-component and a velocity component V_z = 0, the curvature may be ignored and the problem examined as being in the plane x, y.

Let us assume that along z all equations are averaged, i.e., any value a is its average value $(1/\delta)\int_0^\delta a dz$ and in the equations $\partial/\partial z = 0$.

It should be noted that in the boundary layer, where there exist high velocity gradients $\partial V_x / \partial z$ and $\partial V_y / \partial z$, the products which will be encountered in the future $H_z \partial V_x / \partial z$ and $H_z \partial V_y / \partial z$ may not be small, although they drop out in this averaging with respect to z. In the identity

$$\mathrm{curl}\,(\mathbf{V} \times \mathbf{H}) = (\mathbf{H}\nabla)\,\mathbf{V} - (\mathbf{V}\nabla)\,\mathbf{H} + \mathbf{V}\nabla \cdot \mathbf{H} - \mathbf{H}\nabla \cdot \mathbf{V}$$

on the strength of (3) and $\partial / \partial z = 0$ in our case only the second term (convective derivative of H) and the fourth term remain, hence for a plane problem we have from (2)

$$\frac{\partial j_y}{\partial x} - \frac{\partial j_x}{\partial y} = -\mu\sigma\left(\frac{\partial H_z}{\partial t} + \partial\frac{V_x H_z}{\partial x} + \frac{\partial V_y H_z}{\partial y}\right) \tag{11}$$

$$j_x = \frac{\partial H_z}{\partial y} \tag{12}$$

$$j_y = -\frac{\partial H_z}{\partial x} \tag{13}$$

The induction equation takes the form

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = \mu\sigma\left(\frac{\partial H_z}{\partial t} + \partial\frac{V_x H_z}{\partial x} + \frac{\partial H_z V_y}{\partial y}\right) \tag{14}$$

Following the generally accepted representation [12], let us assume that

$$H_z = H + h \tag{15}$$

where

$$H = \dot{H}e^{i(\omega t - ax)} = H_m e^{i(\omega t - ax)} \tag{16}$$

Here H is the external field and h the field of the currents in the gas.

Since the field H is created by currents outside the channel,

$$\mathbf{curl}\ H = 0 \tag{17}$$

and Equation (1) takes the form

$$\mathbf{curl}\, h = j \tag{1'}$$

Since it is assumed that only the z-components of the magnetic field are present, Expression (16) does not satisfy Equation (17). In fact, there are the components H_x and H_y, but they are small due to the smallness of the gap δ.

A determination of the magnitude of H_x may be made from the equation of the y-component curl H

$$\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} = 0 \tag{18}$$

Taking H_m as the scale of the z-component of the field, and the

scale length as one half of the pole separation, where the field increases from 0 to H_m, and the magnitude of the gap as the scale along z, we obtain

$$\Delta H_x \sim H_m 2\delta / \tau \tag{19}$$

since $\delta \ll \tau$, it happens that $\Delta H_x \ll H_m$.

Since it was assumed that walls of the channel have infinite permeability (in actuality, greater by 2 to 3 orders of magnitude than in a vacuum), the walls of the channel are surfaces of an even magnetic potential and, therefore, $H_x = 0$ on the walls. Hence it is apparent that there can be no essential value of the component of the external field H_x in the channel.

Since $\delta \ll l$, the same is true of the y-components. This evaluation is made similarly in cylindrical coordinates

$$H_\theta + r\frac{\partial H_\theta}{\partial r} - \frac{\partial H_r}{\partial \theta} = 0$$

$$H_r \sim H_m, \qquad \partial\theta \sim \frac{\tau}{2r}, \qquad \partial r \sim \delta$$

$$\Delta H_\theta \sim H_m \frac{2r\delta}{\tau(r+\delta)} \tag{20}$$

Since $\delta \ll r$, H_y has an order of $(2\delta/\tau)H_m$.

These evaluations show with what accuracy one may assume the absence of the two components of the external field. (The distribution of the magnetic field in the gap at an arbitrary relationship between δ and τ has been examined in detail by Blake [13].)

The external field H has the form of a traveling wave, therefore, the currents in the gas and their magnetic field h also have the form of a wave of the same frequency as the external field, but with a different initial phase

$$h = \dot{h}e^{i(\omega t - \alpha x)} = h_m e^{i(\omega t - \alpha x + \psi)} \tag{21}$$

Substituting (16) and (21) into (14), we obtain the final form of the induction equation of our problem

$$\frac{d^2\dot{h}}{dy^2} - (\alpha^2 + i\mu\sigma\omega s)\dot{h} - \mu\sigma\frac{d}{dy}(\dot{h}V_y) = \mu\sigma\left[i\omega s\dot{H} + \frac{d}{dy}(\dot{H}V_y)\right] \tag{22}$$

(The term $H_z(\partial V_x/\partial x$ was omitted in connection with the consequent determination with respect to x.)

The slippage

$$s = \frac{\omega - \alpha V_x}{\omega} \tag{23}$$

is a variable which is a function of the peripheral velocity of the gas flow.

From a comparison of Equation (22) with Equation (2) in Vol'dek's work [14], the fundamental difference between this problem and the theory of ordinary electrical apparatus with a massive rotor or the theory of induction pumps in apparent. In ordinary machines, the medium moving in a traveling field has a velocity which is the same at all points and, therefore, s = const.

In our case s is a variable which must be determined using hydrodynamics equations.

In addition, in our case the component of the velocity V_y acts, but is absent in the usual machines.

At s = const and V_y = 0, Equation (22) coincides with Vol'dek's [14].

Equation (22) links complex values and is essentially two equations, which determine the modulus and argument of the complex amplitude of the natural field or it real and imaginary part.

The equations of the hydrodynamics of plane flow has the form

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho V_x}{\partial x} + \frac{1}{\delta}\frac{\partial(\rho V_y\delta)}{\partial y} = 0 \tag{24}$$

$$\rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} = J j_y \mu H_z - \frac{\partial p}{\partial x} + f_{Bx} \tag{25}$$

$$\rho \frac{\partial V_y}{\partial t} + \rho V_x \frac{\partial V_y}{\partial x} + \rho V_y \frac{\partial V_y}{\partial y} = J j_x \mu H_z - \frac{\partial p}{\partial y} + f_{By} \tag{26}$$

$$\rho \left(\frac{\partial i_0}{\partial t} + V_x \frac{\partial i_0}{\partial x} + V_y \frac{\partial i_0}{\partial y} \right) =$$

$$= \frac{\partial p}{\partial t} + A\left(Q - \frac{2q}{\delta}\right) + J j_y \mu H_z \left(\frac{j_y}{\sigma \mu H_z} + V_x\right) + J j_x \mu H_z \left(\frac{j_x}{\sigma \mu H_z} - V_y\right) \tag{27}$$

Since averaging is carried out with respect to z, the forces of viscosity may be written through the coefficient of surface friction $\zeta = 2\tau_w / \rho V^2$

$$f_{Bx} - \zeta \frac{\rho V_x |V|}{\delta}, \qquad f_{By} - \zeta \frac{\rho V_y |V|}{\delta} \tag{28}$$

Since $V_z = 0$, the equation of the z-component of the momentum is transformed into an equation of the equilibrium between the pressure gradient, centrifugal force and the component of the ESF

$$\frac{dp}{dz} = \rho \frac{V_x^2}{r} + J (j_x H_y - j_y H_x) \tag{29}$$

This equation drops out of the system but it may impose limits on the averaging with respect to z since a change in pressure with respect to z leads to a change in gas density and electrical conductivity.

In order to describe stationary flow in a channel with a circular cross section let us average the values with respect to x and t. Since the gap is a smooth function of y and not of x, all values in Equations (24) through (27) can be functions of x and t only owing to the fact that the external field depends upon their arguments. Therefore the dependence of all values upon x and t will be periodic and the average value of the derivatives with respect to x and t must equal zero.

In this it is assumed that the average product equals the product of the averaged cofactors.

The average value of the x-component of the ESF has the form

$$\frac{\omega}{2\pi}\frac{1}{2\tau}\int_0^{\frac{2\pi}{\omega}}\int_0^{2\tau} Jj_y\mu\,(H+h)\,dt\,dx = \frac{1}{2}J\mu\alpha H_m h_m \sin\psi \tag{30}$$

The current density $j_y = i\alpha h$, and after averaging the product $j_y h$ gives zero, since multiplication by $\underline{i}$ means a phase shift of $\pi/2$ and the cofactors become orthogonal functions. Hence it is apparent that the natural field does not contribute to the average x-component of the ESF.

The average y-component of the ESF has the form

$$\frac{\omega}{2\pi}\frac{1}{2\tau}\int_0^{\frac{2\pi}{\omega}}\int_0^{2\tau} Jj_x\mu\,(H+h)\,dx\,dt = \frac{1}{2}J\mu\,\left[H_m\cos\psi + h_m\right]\frac{dh_m}{dy} - $$
$$-H_m h_m \sin\psi\,\frac{d\psi}{dy} \tag{31}$$

The averaged hydrodynamic equations have the form

$$\rho V_y\delta = G \tag{32}$$
$$\rho V_y\frac{dV_y}{dy} + \frac{dp}{dy} = -\frac{J\mu}{2}\left[(H_m + h_r)\frac{dh_r}{dy} + h_a\frac{dh_a}{dy}\right] - \zeta\rho\,\frac{V_y\sqrt{V_x^2+V_y^2}}{\delta} \tag{33}$$
$$\rho V_y\frac{dV_x}{dy} = -\frac{1}{2}J\mu\alpha H_m h_a - \zeta\,\frac{\rho V_x\sqrt{V_x^2+V_y^2}}{\delta} \tag{34}$$
$$\frac{1}{J}\rho V_y\frac{d}{dy}\left(C_pT + \frac{V_x^2+V_y^2}{2}\right) = \frac{J}{2\sigma}\left[\alpha^2\left(h_r^2+h_a^2\right) + \left(\frac{dh_r}{dy}\right)^2 + \left(\frac{dh_a}{dy}\right)^2\right] -$$
$$-J\frac{\mu}{2}\left\{\alpha V_x H_m h_a + V_y\left[\left(H_m + h_r\right)\frac{dh_r}{dy} + h_a\frac{dh_a}{dy}\right]\right\} + \frac{A}{J}\left(Q - \frac{q}{\delta}\right) \tag{35}$$

Here (32) is the generally accepted form of an approximate notation of the continuity equation for long and smoothly defined channels.

The items in the third term of the right-hand part of (35) heat liberation and heat exchange--are determined by equations of chemical or nuclear kinetics and the theory of heat transfer. They may be assumed constant in the first approximation.

The work of the friction forces does not change the over-all enthalpy and therefore drops out of the equation of energy averaged

with respect to z.

The external field is connected with the magnetizing current by the relationship

$$H\delta = F = Jw \tag{36}$$

This expression follows from the law of total current at infinite permeability of the channel walls. Since the magnetizing current does not vary along the magnetic circuit, F is not a function of y.

In an external electrical curcuit with constant voltage with respect to amplitude, the separation of energy manifests itself in the appearance of an active current I_a. The active current shifts in phase relative to the magnetizing reactive current

$$I_a = I_r \cot\varphi$$

The boundary conditions of the problem are:

$$1)\ \text{At}\quad y = -b,\quad y = l + c\quad j_y = h = 0 \tag{37}$$
$$2)\ \text{At}\quad y = -b\quad V = V_1,\quad p = p_1,\quad T = T_1 \tag{38}$$

Equations (22) and (36) require special consideration in the regions -b < y < 0 and l > y > l + c, i.e., in the regions of the so-called "frontal parts" of the rotor, there where there are no magnetic-circuit walls of the channel and H = 0. In these regions the currents in the gas create considerably weaker magnetic fields than in the retion 0 < y < l, i.e., between the magnetic-circuit walls, where a larger part of the length of a closed magnetic line of force passes in the wall, without reluctance. Therefore, one may speak about an increased reluctance and some equivalent decreased permeability μ_{equ}, which replaces μ in the induction equation.

In one limited case in order to find the currents in the "frontal parts" it may be assumed that $\mu = 0$ in Equation (22). Then, to deter-

mine the currents the Laplace formula is obtained, which describes the currents in a noninductive medium which has only resistance (ohmic).

In another limited case, holding μ_{equ} as at 0 y $\underline{1}$, it may be assumed that the channel has magnetic-circuit walls in the frontal parts, but H = 0.

Henceforth, in defining the problem more accurately, it will be advantageous to examine the distribution of the field in the plane x, y, to seek the "coefficient of decrease" in μ, and to obtain a solutions of equations at the boundaries of the regions, i.e., at y = 0 and y = $\underline{1}$, must be combined from the condition of continuity of the currents.

Equations (22), (23), (32) through (36) and (10) make up the fundamental system of equations of a MHD machine with a rotating field, operating as a generator ($\psi > \pi/2$) or as a motor ($\psi < \pi/2$). In the system there are 9 equations (Equation (22) is 2 scalar equations since $\dot{h} = h_r + ih_a$, $h_a = h_m \sin \psi$) and 10 unknowns

$$h_a,\ h_r,\ s,\ \dot{H} = H_m,\ V_x,\ V_y,\ p,\ \rho,\ T,\ \delta$$

Still another equation may be included in the system, one which expressed an additional condition imposed on the flow, for example, $\delta = \delta$ (y) (in an individual case δ = const), T = const, V_y = const, etc.

The system can be reduced to a single resolving equation of the seventh order, for which seven boundary and initial conditions follow from (37) and (38).

As a result of the solutions of the equations of the system, the unknown functions h(y) and $H_m(y)$ are obtained. Through them all characteristics of the machine are expressed in full.

The power factor cos φ is determined by

$$\cot\varphi = \frac{\int_0^l h_a dy}{\int_0^l (H_m + h_r) dy}.$$

The active power is

$$P_a = 3UI\cos\varphi$$

where H_m is connected with the phase voltage of the circuit

$$U = \frac{2}{\pi}\mu\tau\omega w\sqrt{\left[\int_0^l (H_m + h_r) dy\right]^2 + \left(\int_0^l h_a dy\right)^2}$$

The minimum magnetizing current I is determined by a given phase voltage U and the impedance of the stator winding in the absence of liquid or gas flow (dry run).

An evaluation of the quality of a MHD machine, as a converter of energy (not treating the thermodynamic cycle in full), can be easily made using the following three ratios:

1) stator efficiency

$$\eta_{st} = \frac{P_a - P_\Sigma}{P_a}$$

where P_Σ is the sum of losses in the stator (power consumption) to magnetic polarity reversal and Joule effect. This efficiency is no way different from the usual for electric machines.

2) Turbine, or adiabatic, efficiency

$$\eta_T = \frac{JP_a}{2\pi r\, G\left\{\frac{k}{k-1} RT_{01}\left[1 - \left(\frac{p_{02}}{p_{01}}\right)^{\frac{k-1}{k}}\right]\right\}}$$

is the ratio of generated active power to the isentropic work of widening in the pressure interval used to obtain the active power P_a.

3) The share of converted energy (in the absence of heat liberation

$$\eta_{tr} = \frac{JP_a}{2\pi r G i_{01}}$$

The flow of an incompressible fluid. In order to investigate the fundamental, determining effect--the connection between the induction equation and the dynamics equation, i.e., the connection between the natural field and velocity distribution--it is advantageous to examine a more simple case, that of the flow of an incompressible fluid.

Apart from the separate practical interest connected with applications to liquid-metal and electrolyte flows, this solution also gives a representation of the qualitative flow pattern of a compressible fluid (gas) at subsonic speeds.

For simplicity let us ignore friction and let δ = const. Under these conditions the energy equation drops out of the system, $V_y =$ = const, and the system takes form

$$\frac{ds}{dy} = k_1 H_m h_m \sin\psi \tag{39}$$

$$\frac{d^2\dot{h}}{dy^2} - (\alpha^2 + ik_2 s)\,\dot{h} - k_3\frac{d\dot{h}}{dy} = ik_2 H_m s \tag{40}$$

$$k_1 = -\frac{1}{2} J\frac{\mu\alpha^2}{m\omega}, \qquad k_2 = \mu\sigma\omega, \qquad k_3 = \mu\sigma V_y \tag{41}$$

Bearing in mind that $\dot{h} = h_r + ih_a$, where $h_a = h_m \sin\psi$, we obtain simple equations whose left-hand members are linear and constant coefficients

$$\frac{d^2h_r}{dy^2} - k_3\frac{dh_r}{dy} - \alpha^2 h_r = -\frac{k_2}{k_1 H_m}\frac{ds^2}{dy} \tag{42}$$

$$\frac{d^3s}{dy^3} - k_3\frac{d^2s}{dy^2} - \alpha^2\frac{ds}{dy} - k_1k_2H_m^2 s = k_1k_2H_m s h_r \tag{43}$$

Having determined h_r from (43) and substituted it into (42), a single resolving equation of the fifth order may be obtained for finding s, which immediately gives the velocity distribution $V_x(y)$ and the form of the current line of the fluid.

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